

The Family Problem

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Abstract

We all know that in our family of particle physics we have three generations but still don't know why - the so-called "family problem". On other hand, in view of the masses and oscillations, the neutrinos now present some basic difficulty in the Standard Model. In this note, I propose that on top of the $SU_c(3) \times SU(2) \times U(1)$ standard model there is an $SU_f(3)$ extension - a simple $SU_c(3) \times SU(2) \times U(1) \times SU_f(3)$ extended standard model. The family gauge bosons (familons) are massive through the so-called "colored" Higgs mechanism while the remaining Higgs particles are also massive. The three neutrinos, the electron-like, muon-like, and tau-like neutrinos, form the basic family triplets. Hopefully all the couplings to the "visible" matter are through the neutrinos, explaining why dark matter (25 %) is more than visible matter (5 %) in our Universe.

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1 Introduction

More than twenty years ago I was curious by the absence of the Higgs mechanism in the strong interactions but not in the weak interaction sector[1] - a question still remains unanswered till today. A renormalizable gauge theory that does not have to be massless is already reputed by 't Hooft and others, for the standard model. Maybe our question should be whether the electromagnetism would be massless.

Another clue comes from neutrinos - they are neutral, massive and mixing/oscillating. These particles are barely "visible" in the Particle Table. Maybe these are avenues that connect to those unknowns, particularly the dark matter in the Universe.

2 A Proposal

If we think of the role of gauge theories in quantum field theory, we still have to recognize its unique and important role. If the standard model is missing something, a gauge theory sector would be one at the first guess. On the other hand, in the standard model there are three generations of quarks and leptons. But why? It seems to be a first loose point for the standard model. So, let's assume that there is an $SU_f(3)$ gauge theory associated with the story.

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For this $SU_f(3)$ gauge theory, the triplet $(\nu_\tau, \nu_\mu, \nu_e)$ serves as a basic connection. It is likely that only the left-handed components are relevant, but we don't have to worry in this paper. Note the up-side-down position used here because of the way I wrote the positions of the nonzero vacuum expectation values.

The masses of the neutrino triplet come from the coupling to some Higgs field - a pair of complex scalar triplet, as worked out in the previous publication[1]. Hereafter I ignore the "radiative" corrections due to gauge bosons. In this case, the eight components of the Higgs triplets are absorbed by the eight gauge fields through the "family" Higgs mechanism via spontaneous symmetry breaking, while the remaining four become massive Higgs particles. (In the previous application, it was referred to "colored Higgs Mechanism"[1].)

This is the basic framework. The standard model is the gauge theory based on the group $SU_c(3) \times SU(2) \times U(1)$. Now the simple extension is that based on $SU_c(3) \times SU(2) \times U(1) \times SU_f(3)$.

We may write our "new" basic elements as follows. Denote the eight family gauge fields (familons) as $F_\mu^a(x)$. Define $F_{\mu\nu}^a \equiv \partial_\mu F_\nu^a - \partial_\nu F_\mu^a + \kappa f_{abc} F_\mu^b F_\nu^c$. Then we have[2]

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a. \quad (1)$$

One way to describe the nonabelian nature of the gauge theory is to add the Fadde'ev-Popov ghost fields

$$\mathcal{L}_{eff} = \mathcal{L} - \partial_\mu \phi^a(x) D_\mu \phi^a(x), \quad (2)$$

with $D_\mu \phi^a \equiv \partial_\mu \phi^a + \kappa f_{abc} F_\mu^b \phi^c$.

The neutrino triplet $\Psi(x)$ is

$$\mathcal{L}_f = -\bar{\Psi} \gamma_\mu D_\mu \Psi, \quad (3)$$

with $D_\mu \equiv \partial_\mu - i \frac{\kappa}{2} \lambda^a F_\mu^a(x)$. Just like a (triple) Dirac field.

The family Higgs mechanism is accomplished by a pair of complex scalar triplets. Under $SU_f(3)$, they transform into the specific forms in the U-gauge:

$$\begin{aligned} \Phi'_+ &= \exp\{i \frac{\lambda_a}{2} \xi_a^0\} \{u_+ + \rho_+, v_+ + \eta_+, 0\}, \\ \Phi'_- &= \exp\{i \frac{\lambda_a}{2} \xi_a^0\} \{u_- + \rho_-, v_- + \eta_-, 0\}. \end{aligned} \quad (4)$$

We could work out the kinetic terms:

$$\mathcal{L}_{scalar} = -(D_\mu \Phi_+)^{\dagger} D_\mu \Phi_+ - (D_\mu \Phi_-)^{\dagger} D_\mu \Phi_- - V_\Phi, \quad (5)$$

such that, by means of choosing,

$$u_+ = v_- = v \cos\theta, \quad v_+ = -u_- = v \sin\theta, \quad (6)$$

we find, for the familons,

$$\begin{aligned} M_1 = M_2 = M_3 &= \kappa v, \quad M_8 = \frac{\kappa v}{\sqrt{3}}, \\ M_{4,5,6,7} &= \frac{\kappa v}{\sqrt{2}}. \end{aligned} \quad (7)$$

That is, the eight gauge bosons all become massive. On the other hand, by choosing

$$V_\Phi = \frac{\mu^2}{2}(\Phi_+^\dagger \Phi_+ + \Phi_-^\dagger \Phi_-) + \frac{\lambda}{4}\{(\Phi_+^\dagger \Phi_+)^2 + (\Phi_-^\dagger \Phi_-)^2 + 2(\Phi_+^\dagger \Phi_-)(\Phi_-^\dagger \Phi_+)\}, \quad (8)$$

we find that the remaining four (Higgs) particles are massive (with $\mu^2 < 0$, we have $v^2 = -\mu^2/\lambda > 0$).

The other important point is the coupling between the neutrino triplet and the family Higgs triplets:

$$\alpha \bar{\Psi} \times (\Phi_+ + \epsilon \Phi_-) \cdot \Psi, \quad (9)$$

resulting a mass matrix which is off diagonal (but is perfectly acceptable).

What is surprising about our model? There is no unwanted massless particle - so, no disaster anticipated. It is the renormalizable extension of the standard model idea. Coming back to the neutrino sector, we now introduce the mass terms in a renormalizable way (with the help from $SU_f(3)$ gauge theory) - previously a headache problem in the old-day Standard model. Furthermore, there is no major modification of the original Standard Model.

3 Discussions

Our life during the next stage seems to be rather difficult. Neutrinos, albeit abundant, are very elusive. We use neutrinos as the basic bridge to construct the $SU_f(3)$ gauge theory for the family in the building blocks of matter. If the only coupling has to go through neutrinos, then the detection (from the visible side of the matter) would be extremely difficult. Of course, we should look for the potential couplings to other sectors such as quarks or charge leptons.

In the early universe, the temperature could be as high as that for the familons such that the Universe could be populated with these (interacting) particles - just like that for QCD. In other words, our Universe would be full of these particles as the dark matter - at this point, it is believed that our Universe has 25% in dark matter while only 5% in visible matter.

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References

- [1] W-Y. P. Hwang, Phys. Rev. **D32**, 824 (1985) on the "colored Higgs mechanism".
- [2] For notations, see T-Y. Wu and W-Y. Pauchy Hwang, *Relativistic Quantum Mechanics and Quantum Fields* (World Scientific, Singapore, 1991).